MathCastle

Grade 6

Complete course booklet • 11 topics

Contents

1 Ratios and Rates

Comparing two quantities

1.1 Rates as Ratios

The Big Idea

A **ratio** compares two amounts. A **unit rate** is a ratio with 1 in the second slot, which makes scaling up easy: just multiply.

Scaling a recipe

If 12 grams of sugar go with one cup of flour, then 5 cups need $12 \times 5 = 60$ grams.

1.2 In Plain Terms & More Examples

In plain terms

A ratio compares two amounts. A unit rate rewrites it with 1 in the second slot, which makes scaling as simple as multiplying.

A pay rate

If \$60 is earned in 5 hours, the unit rate is $60 \div 5 = \$12$ per hour.

Scaling a ratio

The ratio $2 : 3$ scaled up by 4 becomes $8 : 12$.

2 Percent

Per hundred

2.1 Percent Means Out of 100

The Big Idea

"Percent" literally means **per hundred**, so $p\%$ is the fraction $\frac{p}{100}$. Finding a percent of a number is just multiplying by that fraction.

(diagram in the online version)

Finding 25% of 80

$$\frac{25}{100} \times 80 = \frac{1}{4} \times 80 = 20.$$

2.2 In Plain Terms & More Examples

In plain terms

A percent is a fraction out of 100. To find a percent of a number, turn it into that fraction and multiply.

Finding 10% of 50

$$\frac{10}{100} \times 50 = \frac{1}{10} \times 50 = 5.$$

Finding 75% of 40

$$\frac{75}{100} \times 40 = \frac{3}{4} \times 40 = 30.$$

A multi-step problem

A \$24 shirt is 25% off, and you also have a \$2 coupon. Step 1: 25% of 24 is 6, so the sale price is $24 - 6 = 18$. Step 2: the coupon brings it to $18 - 2 = \$16$.

3 Dividing Fractions

Multiply by the reciprocal

3.1 Why Flip and Multiply

The Big Idea

Dividing asks "how many of these fit?" Dividing by $\frac{c}{d}$ is the same as multiplying by $\frac{d}{c}$, because that undoes the division.

(diagram in the online version)

Finding $\frac{1}{2} \div \frac{1}{4}$

Flip and multiply: $\frac{1}{2} \times \frac{4}{1} = 2$. And indeed two quarters fit inside a half.

3.2 In Plain Terms & More Examples

In plain terms

To divide by a fraction, flip it (its reciprocal) and multiply. Dividing asks how many of the second fraction fit inside the first.

Finding $\frac{3}{4} \div \frac{1}{2}$

Flip and multiply: $\frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2}$.

Finding $\frac{2}{3} \div \frac{1}{6}$

Flip and multiply: $\frac{2}{3} \times \frac{6}{1} = \frac{12}{3} = 4$.

4 Negative Numbers

Numbers below zero

4.1 Both Sides of Zero

The Big Idea

Negative numbers continue the number line to the **left** of zero. Adding still means moving right, and subtracting means moving left — the rule never changes.

(diagram in the online version)

Finding $-6 + 4$

Start at -6 and move 4 to the right, landing on -2.

4.2 In Plain Terms & More Examples

In plain terms

Negatives are the numbers left of zero. Adding still moves right on the number line; subtracting still moves left.

Finding $-3+7$

Start at -3 and move 7 right, landing on 4.

Finding $2-5$

Start at 2 and move 5 left, landing on -3.

5 Absolute Value

Distance from zero

5.1 Distance Is Never Negative

The Big Idea

$|x|$ is how **far** x is from zero, ignoring direction. Distance cannot be negative, so the answer is never negative.

(diagram in the online version)

Why $|-7|=7$

-7 sits seven steps from zero, so its absolute value is 7 — the same distance as $+7$.

5.2 In Plain Terms & More Examples

In plain terms

Absolute value is distance from zero, so it is never negative — just drop the sign.

Finding $|-12|$

It is 12 steps from zero, so $|-12|=12$.

Finding $|8|$

It is already positive: $|8| = 8$.

6 Expressions

Letters standing for numbers

6.1 Substituting a Value

The Big Idea

A letter holds a number we have not chosen yet. To **evaluate**, replace the letter with its value and follow the order of operations.

Evaluating $3x + 5$ at $x = 4$

Substitute: $3(4) + 5 = 12 + 5 = 17$.

6.2 In Plain Terms & More Examples

In plain terms

A letter stands for a number. To evaluate, swap in the value and follow the order of operations.

Evaluating $2x + 7$ at $x = 5$

Substitute: $2(5) + 7 = 10 + 7 = 17$.

Evaluating $5a - 3$ at $a = 4$

Substitute: $5(4) - 3 = 20 - 3 = 17$.

7 One-Step Equations

Undoing what was done to x

7.1 Keep the Balance

The Big Idea

An equation is a balance. Whatever you do to one side you must do to the other, and you undo an operation with its **opposite**.

Solving $4x = 28$

x was multiplied by 4, so divide both sides by 4: $x = 7$. Check: $4 \times 7 = 28$.

7.2 In Plain Terms & More Examples

In plain terms

Undo whatever was done to the letter using the opposite operation — an **inverse operation** — doing the same thing to both sides so the balance holds (the **properties of equality**).

Solving $x + 9 = 15$

Subtract 9 from both sides: $x = 6$. Check: $6 + 9 = 15$.

Solving $\frac{x}{3} = 4$

Multiply both sides by 3: $x = 12$. Check: $12 \div 3 = 4$.

8 Area of Triangles

Half of a rectangle

8.1 Where the Half Comes From

The Big Idea

Any triangle is exactly **half** of a rectangle with the same base and height — that is where the $\frac{1}{2}$ in the formula comes from.

(diagram in the online version)

The formula

Area $= \frac{1}{2}bh$, where h is measured **perpendicular** to the base, not along a slanted side.

Base 10, height 6

Area $= \dfrac{1}{2}(10)(6) = 30$ square units.

8.2 In Plain Terms & More Examples

In plain terms

A triangle is half a rectangle with the same base and height, so its area is $\frac{1}{2} \times \text{base} \times \text{height}$.

Base 8, height 5

Area $= \dfrac{1}{2}(8)(5) = 20$ square units.

Base 12, height 3

Area $= \dfrac{1}{2}(12)(3) = 18$ square units.

9 Surface Area and Volume

Wrapping versus filling

9.1 Two Different Measures

The Big Idea

Volume fills the inside (cubic units). **Surface area** wraps the outside (square units) — add up the six faces, which come in matching pairs.

(diagram in the online version)

A 4 by 3 by 2 box

Volume $= 4 \cdot 3 \cdot 2 = 24$. Surface area $= 2(12 + 6 + 8) = 52$.

9.2 In Plain Terms & More Examples

In plain terms

Volume fills the inside (cubic units); surface area covers the outside (square units) — add the areas of all six faces.

A cube of side 3

Volume $= 3^3 = 27$. Surface area $= 6 \times 3^2 = 54$.

A 5 by 2 by 2 box

Volume $= 5 \cdot 2 \cdot 2 = 20$. Surface area $= 2(10 + 4 + 10) = 48$.

10 Statistics

Mean, median and spread

10.1 Two Kinds of Middle

The Big Idea

The **mean** shares the total out equally. The **median** is the value in the middle once the data is in order. One outlier drags the mean but barely moves the median.

(diagram in the online version)

Even-sized sets

With an even count there is no single middle value, so the median is the **average of the two middle** numbers.

10.2 In Plain Terms & More Examples

In plain terms

The mean shares the total out equally; the median is the middle value once the numbers are in order.

Mean of $4, 6, 8, 10$

Total $= 28$, and $28 \div 4 = 7$, so the mean is 7.

Median of $3, 5, 9$

In order, the middle value is 5.

11 Quadrants

The four regions of the plane

11.1 Reading the Signs

The Big Idea

The axes cut the plane into four quadrants, numbered anticlockwise from the top right. The **signs** of x and y tell you which one you are in.

(diagram in the online version)

The pattern

I is $(+, +)$, II is $(-, +)$, III is $(-, -)$, IV is $(+, -)$.

11.2 In Plain Terms & More Examples

In plain terms

The signs of x and y tell you the quadrant: quadrants are numbered anticlockwise from the top right.

Where is $(-4, -1)$?

Both are negative, so it is in quadrant III.

Where is $(5, -2)$?

x is positive and y is negative, so it is in quadrant IV.